CALCULATION OF HEAT TRANSFER IN NOZZLES

B. P. Mironov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 4, pp. 116–120, 1966

The author proposes a simplified method for calculating the heat transfer in subsonic and supersonic nozzles. The method is based on a solution of the energy equation using relative heat transfer laws.

The intensity of heat transfer in nozzles depends on many factors such, for example, as gas flowrate, geometrical dimensions, compressibility, nonisothermicity, length of preconnected section, roughness, chemical reactions, etc.

Formulas for calculating the heat transfer in nozzles were obtained in experimental studies [1–5]. In these formulas the basic parameters are the gas flowrate and the nozzle dimensions. The effect of other factors is taken into account somewhat arbitrarily, for example, by selecting a corresponding characteristic temperature, introducing correction factors, and so on.

More effective results can be obtained by combining experimental data with theoretical methods that take into account the longitudinal development of the thermal boundary layer. In [6, 7] Bartz gives an approximate theoretical solution of the problem of heat transfer in nozzles. In this solution the friction law obtained for an incompressible gas is extended to include compressibility by introducing an "arithmetic mean" characteristic temperature. In [8] Repik and Chekalin propose a calculation method based on a certain effective length which takes into account the previous history of development of the boundary layer. The present paper examines a simplified method of determining the heat transfer in nozzles based on a solution of the energy equation using relative heat transfer laws.

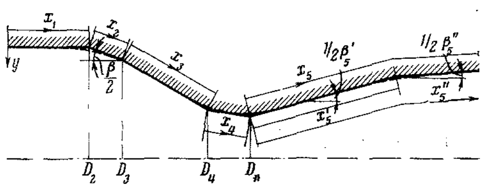

Fig. 1

For axisymmetric flow in the absence of a transverse mass flux the equations of the thermal boundary layer in integral form may conveniently by written as [9]

$$\frac{dR_T^{**}}{dx} + R_T^{**}\left(\frac{1}{\Delta T}\frac{d\Delta T}{dx} + \frac{1}{r}\frac{dr}{dx}\right) = R_L S ,$$

$$R_T^{**} = \frac{\delta_T^{**} w_0 \rho_0}{\mu_{00}} ,$$

$$\delta_T^{**} = \int_0^\delta \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{T_w - T^*}{T_w - T_{00}}\right)\left(1 \pm \frac{y}{r}\cos\frac{\beta}{2}\right) dy , \qquad (1)$$

$$\Delta T = T_w^* - T_w, \quad T_w^* = T_0(1 + {}^1\!/_2 n\,(k-1)\,M^2,$$

$$R_L = \frac{w_0 L \rho_0}{\mu_{00}} , \qquad x = \frac{X}{L} ,$$

$$S = \Psi S_0, \qquad S_0 = \frac{B}{2\,(R_T^{**})^m P^{0.75}}\left(\frac{\mu_0}{\mu_{00}}\right)^m . \qquad (2)$$

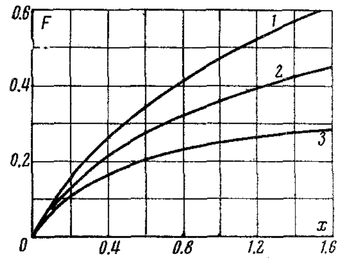

Fig. 2. F(x) as a function of x and k. The curves 1, 2, 3 correspond to the values k = 1.2, 1.4, 1.67.

Here r is the variable nozzle radius, x a coordinate reckoned along the generator, L a certain characteristic length, ρ, w, ρ_0, w_0 the density and velocity in the boundary layer and at its outside edge, n the recovery vactor, k the adiabatic exponent, M the Mach number, μ_0, μ_{00} the values of the dynamic viscosity at the thermodynamic T_0 and stagnation T_{00} temperatures; the subscript w denotes the parameters at the wall; Ψ is a relative quantity representing the ratio of the Stanton number S under the given conditions (i.e., in the presence of compressibility, nonisothermicity, chemical reactions, etc.) to the Stanton number on a flat plate at the same values of R_T^{**}, but in the absence of perturbing factors.

The value of Ψ taking into account the effect of nonisothermicity and compressibility can be found both experimentally and on the basis of the theory of limiting laws. In accordance with [10],

$$\Psi = \left(\frac{2}{\sqrt{T_w/T_w^*} + 1}\right)^2 \left[\frac{\operatorname{arc\,tg} \sqrt{{}^1\!/_2 n\,(k-1)}\,M}{\sqrt{{}^1\!/_2 n\,(k-1)}\,M}\right]^2 . \qquad (3)$$

D. B. Spalding, approximating the formulas presented in [8], has proposed the following expression for Ψ:

$$\Psi = \left[\frac{1}{4}\left\{\left(\frac{T_w}{T_0}\right)^{1/2} + 1\right\}^2 + \frac{1}{6}\, n\, \frac{k-1}{2}\, M_0^2\right]^{-1} . \qquad (4)$$

The integral of Eq. (1) is equal to

$$R_T^{**} = \frac{1}{r\Delta T}\left\{\frac{(1+m)\,B}{2P^{0.75}}\int_0^x \Psi R_L\,(r\Delta T)^{1+m}\left(\frac{\mu_0}{\mu_{00}}\right)^m dx + \right.$$

$$\left. + \left[r\Delta T R_T^{**}\right]_{x=0}^{1+m}\right\}^{\frac{1}{1+m}} . \qquad (5)$$

Having (2), (3), and (5), we can determine the values of the heat transfer coefficients along the length of the nozzle. The validity of this method is confirmed in [9]. However, this calculation is rather laborious.

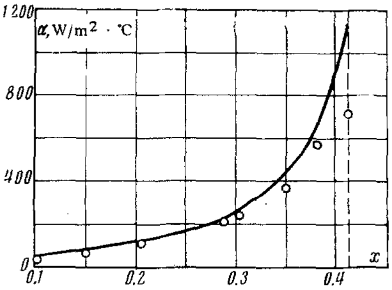

Fig. 3. Comparison of theoretical curve with experimental data of [1] at $\psi = 0.637-0.765$.

Below we will consider certain simplifications of the theoretical method, which make possible a considerable reduction in the volume of calculation. At the same time, the generality of the method is preserved and the accuracy remains good enough for practical purposes.

We will consider a subsonic nozzle. Since in this case the thermodynamic flow temperature varies only slightly along the length of the nozzle, the ratio μ_0/μ_{00} is close to unity. If we introduce a certain mean wall temperature T_W and assume that it is constant along the length of the subsonic part of the nozzle, then the value of Ψ in (5) may also be considered constant. This assumption is perfectly permissible in determining R_T^{**} for finding S_0 from the second of Eqs. (2), since in this equation R_T^{**} is present to the power $m = 0.25$. In order to find S from (2), the expression for Ψ may be taken as a function of x.

Any axisymmetric nozzle contour can be represented as consisting of a series of conic sections (Fig. 1). We take the origin for calculating coordinate x along the generator as shown in Fig. 1, i.e., at the beginning of each conic section. The length scale L for each section

$$L_i = \frac{D^\circ}{\sin {}^1/_2\beta_i}. \tag{6}$$

Here D° is the inlet diameter of the conic section, β_i is its total taper angle ($0 < \beta_i/2 < 90°$).

We express the variation of the parameters along the nozzle in terms of the continuity equation

$$\frac{w_0\rho_0}{w^\circ\rho^\circ} = \frac{D_i^{\circ 2}}{D_i^2}. \tag{7}$$

Here D_i is the variable nozzle diameter. For a convergent nozzle

$$D_i/D_i^0 = 1 - 2x_i \quad (0 < x < 0.5).$$

Equation (5) can now be reduced to the form

$$R_T^{**} = \frac{1}{1-2x_i}\left\{\frac{0.0315}{p^{0.75}}\Psi R_i^\circ\left[1-(1-2x_i)^{0.25}\right] + (R_T^{**})_{x_i=0}^{1.25}\right\}^{0.8},$$
$$R_i^\circ = \frac{w_i^\circ\rho_i^\circ D_i^\circ}{\mu_{00}\sin {}^1/_2\beta_i}. \tag{8}$$

Here the inlet parameters are denoted by the sign °. The values of B and M have been taken as 0.0252 and 0.25 respectively.

The heat transfer coefficients in each conic section are found from the expression

$$\alpha_i = 3600\gamma_0 w_0 c_p S = \frac{3600 G c_p \Psi S_0}{{}^1/_4\pi(1-2x_i)^2 D_i^{\circ 2}}. \tag{9}$$

Here G is the gas flowrate through the nozzle per second, and c_t is the specific heat.

Finally, we have

$$\alpha = \frac{57.8 G c_p \Psi}{(1-2x_i)^2 D_i^{\circ 2}(R_T^{**})^{0.25}p^{0.75}}, \tag{10}$$

where values of R_T^{**} are found from Eq. (8).

We will now consider the supersonic part of the nozzle. For this part it is also possible to assume the constancy of a certain mean wall temperature. At moderate values of $M \leq 3$ we may assume that $(\mu_0/\mu_{00})^{0.25} \sim 1$. For example, for air $(\mu_0/\mu_{00})^{0.25} = 0.9$ at M = 3. In practice, the contour of the supersonic part of the nozzle is often conical or can be sufficiently closely approximated by a single conic section. Here, as distinct from in the subsonic part, the function Ψ varies appreciably with respect to x. However, at T_W = const the factor in Eq. (3) containing the ratio T_W/T_W^* can be taken out of the integrand, since T_W^* scarcely changes along the length of the nozzle. Thus, with account for (7), Eq. (5) for a divergent nozzle becomes

$$R_T^{**} = \frac{1}{1+2x_i}\left[\frac{(1+m)B}{2P^{0.75}}R_*\left(\frac{2}{\sqrt{T_w/T_w^*}+1}\right)^2 \times \int_0^{x_i}\left(\frac{\operatorname{arctg}\sqrt{{}^1/_2 n(k-1)}\,M}{\sqrt{{}^1/_2 n(k-1)}\,M}\right)\left(\frac{D_i}{D_*}\right)^{m-1}dx_i + (R_T^{**})_{x_i=0}^{1+m}\right]^{\frac{1}{1+m}},$$
$$R_* = \frac{w_*\rho_* D_*}{\mu_{00}\sin {}^1/_2\beta_i}. \tag{11}$$

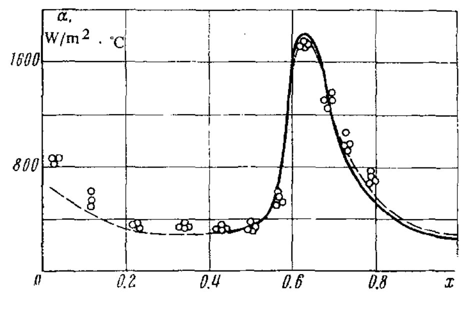

Fig. 4. Comparison of theoretical curves and experimental data [9] for $\psi = 0.580-1.48$ $M \leq 2.9$ (convergent parts approximated by three conical surfaces; continuous curve based on (8), (10), broken curve based on (5)).

Here the nozzle throat parameters are denoted by a an *. For a one-dimensional flow the values of M are uniquely related with the ratio D_i/D_* or with the coordinate x_i. Thus, the integral of Eq. (11) can be evaluated and tabulated. Figure 2 gives values of this integral for different k at n = 0.9. Since $(\mu_0/\mu_{00}) = (T_0/T_{00})^q$, while T_0/T_{00} is a function of the M number and hence of the coordinate x_i, the integral of Eq. (11) can be tabulated without assuming that $(\mu_0/\mu_{00})^m \approx 1$. This is desirable when M > 3 and when the exponent q is known for the conditions in question.

Thus, for determining the heat transfer coefficients in the nozzle we obtain the equations (B = 0.0252, m = 0.25)

$$R_T^{**} = \frac{1}{1+2x_i}\left\{\frac{0.0157}{p^{0.75}} R_* \left(\frac{2}{\sqrt{T_w/T_w^*}+1}\right)^2 F(x_i) + (R_T^{**})^{1.25}_{x_i=0}\right\}^{0.8},$$

$$\alpha = \frac{57.8 G c_p \Psi}{(1+2x_i)^2 D_*^2 (R_T^{**})^{0.25} p^{0.75}} \left(\frac{\mu_0}{\mu_{00}}\right)^{0.25}. \quad (12)$$

Values of $F(x_i)$ are given in Fig. 2; x_i is reckoned from the throat section. The value of R_T^{**} at $x_i = 0$ is known from the calculation for the subsonic part of the nozzle.

When it is necessary to approximate the nozzle contours by several conic sections, the first of Eqs. (12) changes form. In the notation of Fig. 1, the heat transfer is calculated from

$$R_* = R_*' = \frac{w_* \rho_* D_*}{\mu_{00} \sin {}^1\!/_2\beta_5'} \qquad (x_5 = x_5'),$$

$$R_*'' = \frac{w_* \rho_* D_*}{\mu_{00} \sin {}^1\!/_2\beta_5''} \qquad (x_5 > x_5'),$$

$$R_T^{**} = \frac{1}{1+2x_5}\left\{\frac{0.0157}{p^{0.75}} R_*'' \left(\frac{2}{\sqrt{T_w/T_w^*}+1}\right)^2 [F(x_5) - F(x_5')] + [(1+2x_5) R_T^{**}]^{1.25}_{x_5=x_5'}\right\}^{0.8}.$$

In this case, too, the values of x_5 are reckoned from the throat section irrespective of whether the supersonic part of the nozzle consists of one or more conic sections. This is possible thanks to relation (6), as a result of which x_5 is a function only of the ratio of the diameters:

$$x_5 = 0.5\ (D_i/D_* - 1).$$

As a result of these simplifications, equations convenient for calculation are obtained. The labor of calculation can be substantially reduced, if all the cofactors of the equations containing x are tabulated.

In Figs. 3–5 the experimental data and data calculated from Eq. (5) and by the simplified method (Eqs. (8), (10), (12)) are compared. As may be seen from the graphs, the agreement between theory and experiment is quite satisfactory. An exception is formed by Bakirov's data for the throat section only, which are considerably lower than the calculated values.

The relative law of heat transfer Ψ deserves special attention. For the subsonic part of nozzles the effect of the M number on Ψ can be neglected and thus in this zone the quantity Ψ depends almost exclusively on the temperature factor

$$\frac{T_w}{T_w^*} \approx \frac{T_w}{T_0} = \psi.$$

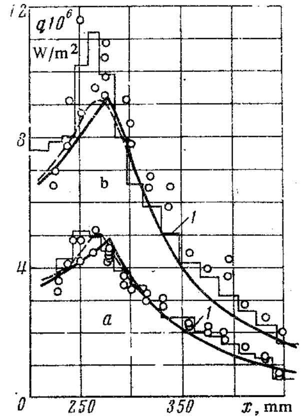

Fig. 5. Comparison of calculations with the experimental data of [4] (convergent part approximated by a single conic section); $\psi \geq 0.225$ $M \leq 2.6$. Curves a, b correspond to experiments at pressures of $6 \cdot 10^6$ N/m^2, $1.38 \cdot 10^6$ N/m^2, the stepped curve 1 corresponds to calorimetric measurements.

In the above examples $\psi < 1$. The published experimental data on the effect of the temperature factor on heat transfer at $\psi < 1$ are rather contradictory, and thus there is still no unanimous opinion about the dependence of Ψ on ψ at $\psi < 1$.

In accordance with the theory of limiting laws, at $\psi < 1$ the function Ψ increases with decrease in ψ. In this case the correction for finite R_T^{**} numbers has an appreciable effect on Ψ only at small values of ψ.

The theoretical and experimental data presented in Figs. 3–5 confirm the regular qualitative and quantitative influence of Ψ on heat transfer at $\psi < 1$. The values of Ψ in (10), (12) for ψ close to unity were calculated from the limit formula (3), while for small values of ψ with account for finite R_T^{**} numbers [8] and under the conditions considered the value of Ψ varied from 1.1 to 1.3. Since in (10), (12) the R_T^{**} number is present to the power 0.25, values of Ψ for determining R_T^{**} from (5) were found from the limit relations (3), (4).

It should be mentioned that theoretical calculations using the function Ψ make it possible to calculate the heat transfer in nozzles under nonstationary conditions. The proposed simplified method can also be extended to the case of variable wall temperature. For this it is necessary to average the wall temperature within the limits of each conic section. We note that this simplified method always permits estimation of the error introduced as compared with the exact method on Eq. (5).

REFERENCES

1. N. U. Bakirov, "Convective heat transfer in convergent and divergent axisymmetric nozzles," Izv. VUZ. Aviatsionnaya Tekhnika, no. 4, 1958.

2. O. A. Saunders and P. H. Calder, "Heat transfer in nozzles at supersonic speeds," Eng., August, no. 8, 1952.

3. W. E. Welsh and A. B. Witte, "Comparison of local heat fluxes found theoretically and experimentally in the chambers of liquid rocket motors," Teploperedacha [Russian translation], no. 1, 1962.

4. A. B. Witte and E. H. Harper, "Experimental investigation of heat transfer in rocket motor nozzles," Raketnaya tekhnika i kosmonavtika [Russian translation], no. 2, 1963.

5. S. Greenfield, "Determination of rocket motor heat transfer coefficients by the transient method," J. Amer. Scien., vol. 18, no. 8, 1951.

6. D. R. Bartz, "An approximate solution of compressible turbulent boundary layer development and convective heat transfer in convergent-divergent nozzles," Trans. ASME, vol. 77, no. 8, 1955.

7. D. R. Bartz, "A simple equation for rapid estimation of rocket nozzle convective heat transfer coefficients," J. Jet Propuls., no. 1, 1957.

8. E. U. Repik and V. E. Chekalin, "Convective heat transfer in supersonic nozzles," Inzh. zh., vol. 2, no. 2, 1962.

9. S. S. Kutateladze and A. I. Leont'ev, Turbulent Boundary Layer of a Compressible Gas [in Russian], Izd. SO AN SSSR, 1962.

10. Collection: Friction and Heat and Mass Transfer in a Turbulent Boundary Layer [in Russian], edited by S. S. Kutateladze, Izd. SO AN SSSR, 1964.

6 July 1964 Novosibirsk